Comment on "Black Holes are Neither Particle Accelerators Nor Dark Matter Probes"

O. B. Zaslavskii

Department of Physics and Technology,

Kharkov V.N. Karazin National University,

*4 Svoboda Square, Kharkov 61022, Ukraine**

The Banados-Silk-West effect consists in the possibility to get infinite energy in the centre of mass frame of two particles colliding near the black hole horizon. According to S. T. McWilliams, Phys. Rev. Lett. **110** (2013) 011102, the energy at infinity of the outcome vanishes because of infinite redshift when the point of collision approaches the horizon. I show that this is not so.

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The effect discovered by Bañados, Silk and West (BSW) states that if two particles collide near the black hole horizon, the energy in their centre of mass frame $E_{c.m.}$ can grow unbound [1]. For astrophysical purposes, it is important to know what can be detected at infinity as the products of collision. Here, there are two kinds of relevant quantities: (i) fluxes from a vicinity of the horizon, (ii) masses m and energies E_∞ of particles. In both cases strong redshift is crucial. Its account in a recent work [5] lead to the conclusions that (i) fluxes vanish due to relativistic dilatation of time, (ii) $E_\infty \to 0$. Conclusion (i) looks reasonable and is essential for estimates of expected fluxes. However, (ii) is incorrect.

Let particles 1 and 2 collide to produce particles 3 (escapes) and 4 (falls into a black hole). Eq. 6 of [5] reads

$$E_\infty = \alpha E_{c.m.} \tag{1}$$

where α is the lapse due to geodesic motion. For a general stationary axially symmetric black hole $\alpha = (u^0)^{-1} = \frac{N^2}{E_\infty - \omega L}$, $\omega = -g_{0\phi}/g_{00}$, N is the lapse function entering the metric. Near the horizon, $N \to 0$, $E_{c.m..} \sim \frac{1}{\sqrt{N}}$ [6] but α oversomes it, so eq. (1) leads to the conclusion that $(E_\infty)_3 \to 0$ for any L (not only for the case $L = 0$ in eq. 7 of [5]). Meanwhile, the

*Electronic address: zaslav@ukr.net

correct formula is

$$(E_{\infty})_3 - m_3 (u_i)_3 \left(u^i\right)_4 \alpha = \alpha E_{loc.} = \alpha m_3 \gamma(3,4), \tag{2}$$

$\gamma(3,4) = -(u_\mu)_3 (u^\mu)_4$ is the Lorentz factor of relative motion, α refers to particle 4, $\mu = 0, i$, $E_{c.m.}^2 = m_3^2 + m_4^2 + 2m_3 m_4 \gamma(3,4)$.

It follows from the geodesic equations and (2) that for the near-horizon collision, $(E_{\infty})_3 \approx \omega_H L_3$, ω_H is the black hole angular velocity. This is in perfect agreement with the previous reuslts [2] - [4]. Thus strong redshift is quite compatible with nonzero $(E_{\infty})_3$. The incorrect conclusion about $(E_{\infty})_3 \to 0$ was based on (i) confusion between $E_{c.m.}$ and $E_{loc.}$ and (ii) omission of kinetic terms in (2). Thus for individual collisions there are no more restrictive bounds than those already obtained in [2] - [4].

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